

# Prediction of the Influence of Residual Stresses to Strength and Lifetime of Process Equipment

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*Using the results obtained before one apply the concept of total specific energy participation,  $P_T$ , to effects superposition in the case of shells loading, taking into account the influence of residual stresses. The obtained results have been applied to superpose the effects by strength and lifetime calculation by considering residual stresses. In calculation the total participation  $P_T$  the following cases of effects superposition there are considered: loading under creep conditions, loading in corroding media, fatigue in a corroding environment, fatigue under creep in a corroding environment.*

*Keywords: pressure vessels, effects superposition, nonlinear materials, residual stresses, energy principle, strength, lifetime, buckling, creep, fatigue, corrosion*

In the process of manufacturing the cylindrical and conical parts of pressure vessels – for example – there are induced residual stresses. Application of heat treatment will generally diminish – though seldom fully eliminate – the residual stresses generated during the various stages of manufacturing a certain part of the vessels.

The possible effect of neglecting these stresses in the design of structures might be the fact that, during service, critical state are exceeded, eventually leading to cracking and sometimes even to vessel destruction.

The problem arising is to answer the question: how does one take into consideration residual stresses when calculating the strength of mechanical structure components?

At present, in a structure loaded with a normal stress  $\sigma$ , determined by process parameters, one often calculates total tensile stress with relation,

$$\sigma_t = \sigma + \sigma_{res} \quad (1)$$

where the residual stress,  $\sigma_{res}$ , is frequently considered equal to the yield strength of the material  $\sigma_{res} = \sigma_y(T)$  under working temperature,  $T$ .

Relation (1) is valid only for the linear behaviour of materials. Most metallic materials behave linearly (within acceptable approximation) under stresses that are lower than the yield strength. This behaviour is represented by Hook's law for normal stresses  $\sigma$ , which may be also applied to shear stresses  $\tau$ , too,

$$\left. \begin{aligned} \sigma &= E \cdot \varepsilon \\ \tau &= G \cdot \gamma \end{aligned} \right\} \quad (2)$$

where:

$E$ ,  $G$  - elongation modulus of elasticity (Young's modulus) and the modulus of transversal elasticity, respectively;

$\varepsilon$  - strain;

$\gamma$  - shear strain

However, as one knows, residual stresses are the outcome of material deformation beyond the yield strength at ( $\sigma > \sigma_y$ ):

- the material behaviour is non-linear, expressed by power law of the form [1]

$$\left. \begin{aligned} \sigma &= M_\sigma \cdot \varepsilon^n \\ \tau &= M_\tau \cdot \gamma^m \end{aligned} \right\} \quad (3)$$

where:

$M_\sigma$ ,  $M_\tau$ ,  $n$  and  $m$  are material constants. Often  $m = n$ . If  $m = n = 1$  relations (3) become relations (2);

- strains are high as compared to the case when  $\sigma \leq \sigma_y$ .

The consequences of these particular loading features beyond the yield strength are:

- one cannot apply Boltzaman's principle of linear superposition, given in this case by relation (1);

- one resorts to the concepts of natural normal stress,  $\sigma_n$ , and natural specific strain,  $\varepsilon_n$  (in Hencky' sense of the term) instead of the engineering concepts,  $\sigma$  and  $\varepsilon$ . The latter are involved in the well known relations [2]:

$$\left. \begin{aligned} \sigma_n &= \sigma \cdot (1 + \varepsilon) \\ \varepsilon_n &= \ln(1 + \varepsilon) \end{aligned} \right\} \quad (4)$$

Based on the above considerations, we shall further put forward some relations and a method of calculating the state of stress taking into account the residual stresses.

## Calculation of strength by considering residual stresses

Generally speaking, the residual stresses may influence the deterioration of materials. The deterioration of materials have been analysed in the papers [3, 4].

## General analysis

We shall further indicate stresses as  $\delta$  and  $\tau$  leaving it up to the designer to choose between dependencies  $\sigma(\varepsilon)$  and  $\tau(\gamma)$  or  $\sigma_n(\varepsilon_n)$  and  $\tau_n(\gamma_n)$ .

The influence of residual stresses upon the state of loading has been highlighted by using the principle of critical energy, according to which the total specific energy participation with respect to the critical state (recently called Jinescu's criterion,  $J_i \equiv P_i$  [5]) determined by the external actions is [1, 6],

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$$P_T = \sum_j \left( \frac{\sigma_j}{\sigma_{j,cr}} \right)^\alpha \cdot \delta_j + \sum_i \left( \frac{\tau_i}{\tau_{i,cr}} \right)^\alpha \cdot \delta_i \quad (5)$$

where one took into consideration the non-linear behaviour of a material according to relations (3) as  $n = m$  and where one wrote  $\alpha = \frac{1}{n} + 1$ . Factor  $\delta_j$  takes the following possible values:

$$\delta_j = \begin{cases} 1, & \text{if } \sigma_j \text{ acts in the sense of the process evolution;} \\ 0, & \text{if } \sigma_j \text{ has no influence upon the process;} \\ -1, & \text{if } \sigma_j \text{ opposes the process.} \end{cases} \quad (6)$$

Analogously, one has to deal with the same kind of problem regarding  $\delta_i$ , with respect to the sense of action  $\tau_i$ .

For a section which does include residual stresses, the total participation is written

$$P_{T,res} = P_T + P_{res} \quad (7)$$

The participation of residual stresses,  $P_{res}$ , is calculated by using the definition of specific energy participation with respect to the critical state [1, 7]

$$P_{res} = \frac{E_{res}}{E_{res,cr}} \cdot \delta_{res} \quad (8)$$

where  $E_{res}$  is the specific energy accumulated in the material because of the residual stress load  $\sigma_{res}$ , while  $E_{res,cr}$  – the critical value of  $E_{res}$ . Factor  $\delta_{res}$  takes values 1; 0 and -1.

In a simple tension test (fig. 1) with  $\sigma_n > \sigma_{ny}$ , segment OAB of stress – strain diagram,  $\sigma_n - \epsilon_n$ , is covered. Unloading until the stresses are annihilated, unfolds along straight-line parallel to elastic behaviour segment OA. Owing to the interaction of material fibres, the unloading will actually unfold until point C has been reached. The body remains deformed with the value corresponding to segment whose corresponding residual stress is equal to segment CD. Upon getting reloaded with total stress  $\sigma_{n1}$ , straight line CB is covered. The behaviour of the material after the unloading of  $BO_1$  is related to the system of coordinating axes  $\sigma_{n1}, \epsilon_{n1}$  instead of  $\sigma_n, \epsilon_n$ . For a new loading, after the introduction of residual stresses, the specific energy  $E_{res}$  corresponds to the triangle area  $O_1CD$  and is

$$E_{res} = \frac{1}{2} \cdot \sigma_{res} \cdot \overline{O_1D} = \frac{\sigma_{res}^2}{2E}$$

where  $E$  is the elongation modulus of elasticity.

Upon reaching the critical state

$$E_{res,cr} = \frac{\sigma_{res,cr}^2}{2E}$$

so that

$$\frac{E_{res}}{E_{res,cr}} = \left( \frac{\sigma_{res}}{\sigma_{res,cr}} \right)^2 \quad (9)$$

Out of relations (7) - (9), it follows that

$$P_{T,res} = P_T + \left( \frac{\sigma_{res}}{\sigma_{res,cr}} \right)^2 \cdot \delta_{res} \quad (10)$$

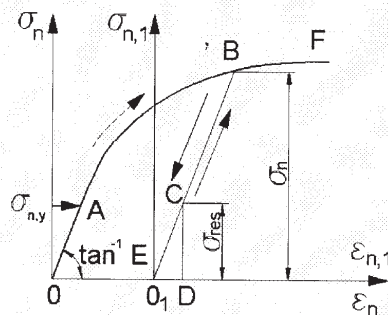


Fig. 1. Dependence  $\sigma_n - \epsilon_n$  when the material is first loaded (OAB) and after a process of loading – unloading – reloading (OAB – BC – CBF)

where  $\sigma_{res,cr}$  is the residual critical stress, which by acting alone, causes the occurrence of the critical state (cracking, failure etc.);

$$\delta_{res} = \begin{cases} 1, & \text{if the residual stresses have the same sign as the stresses determining external loading;} \\ 0, & \text{if the residual stresses do not influence the state of loading;} \\ -1, & \text{if the sign of the residual stresses is contrary to the stresses supplied by external loading.} \end{cases}$$

For example, if the working stresses are tensile stresses, while the residual stresses are compression stresses, then  $\delta_{res} = -1$ .

The loading state is not critical if [1; 6]

$$P_{T,res} \leq P'_{cr} \quad (11)$$

where:

$$P'_{cr} = \begin{cases} P_{cr}, & \text{in processes of isothermal deformation;} \\ P_{cr} - \frac{E_{th}}{E_{cr}}, & \text{in processes of non-isothermal deformational;} \end{cases} \quad (12)$$

$$P_{cr} \in [P_{cr,min}; P_{cr,max}], \text{ with } P_{cr,max} \leq 1.$$

$E_{th}$  is the specific energy dissipated as heat;  
 $E_{cr}^{th}$  - the specific critical energy for the process under analysis.

By considering relation (10), which involves the participation of residual stresses, out of condition (11) one obtains:

$$P_T \leq P_{cr}^* \quad (13)$$

where  $P_T$  is given by relation (5) and

$$P_{cr}^* = P_{cr} - \frac{E_{th}}{E_{cr}} - \left( \frac{\sigma_{res}}{\sigma_{res,cr}} \right)^2 \cdot \delta_{res} \quad (14)$$

The total participation with respect to the allowable state, by considering the residual stress, is

$$P_{T,res}^* = P_T^* + \left( \frac{\sigma_{res}}{\sigma_{res,al}} \right)^2 \cdot \delta_{res} \quad (15)$$

where

$$P_T^* = \sum_j \left( \frac{\sigma_j}{\sigma_{j,al}} \right)^\alpha \cdot \delta_j + \sum_i \left( \frac{\tau_i}{\tau_{i,al}} \right)^\alpha \cdot \delta_i \quad (16)$$

$\sigma_{j,al} = \frac{\sigma_{j,cr}}{c_\sigma}$  and  $\tau_{i,al} = \frac{\tau_{i,cr}}{c_\tau}$ ;  $c_\sigma$  and  $c_\tau$  are the safety coefficients.

The loading state is allowable if [1; 7],

$$P_T^* \leq P_{al}'' \quad (17)$$

where:

$$P_{al}'' = \begin{cases} 1 - \left( \frac{\sigma_{res}}{\sigma_{res,al}} \right)^2 \cdot \delta_{res} & \text{in processes of isothermal} \\ & \text{deformation;} \\ 1 - \frac{E_{dh}}{E_{cr}} - \left( \frac{\sigma_{res}}{\sigma_{res,al}} \right)^2 \cdot \delta_{res} & \text{in processes of non-} \\ & \text{isothermal deformation;} \end{cases} \quad (18)$$

$$\sigma_{res,al} = \frac{\sigma_{res,cr}}{c_\sigma}$$

We shall further deal with some particular cases of uniaxial loading of a material with residual stresses.

#### Loading under creep conditions, involving consideration of residual stresses

In the case of creep loading of a structure ( $T \geq T_{creep}$ ) with residual stresses the time taken to reach failure is reduced. The total energy participation with respect to the critical state, in the case of uniaxial loading under normal stress  $\sigma$ , over duration  $t$ , is written as [1]

$$P_T = \left( \frac{\sigma}{\sigma_{cr}} \right)^\alpha + \frac{t}{t_{cr}}, \quad (19)$$

where  $\sigma_{cr} = \sigma_{cr}(0)$  is the failure strength at temperature  $T \geq T_{creep}$  and  $t = 0$ ;  $t_{cr} = t_{c,0}$  - duration until the end of the stabilised creep zone II in diagramme  $\varepsilon - t$ , (fig. 2).

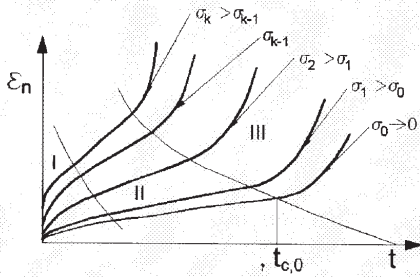


Fig. 2. Creep curve under loading stress  $\sigma_0, \sigma_1, \dots, \sigma_k$  at temperature values higher than creep temperature ( $T > T_{creep}$ )

Out of relation (19), on condition that  $P_T = P_{cr}''$ , one obtains the time to reach the critical state or the lifetime, under creep conditions, in the presence of residual stresses:

$$t_{cr}^* = t_{cr} \cdot \left[ P_{cr}'' - \left( \frac{\sigma}{\sigma_{cr}} \right)^\alpha \right] \quad (20)$$

The time taken to reach the allowable state is obtained from relation (20) and is written as

$$t_{al}^* = t_{al} \cdot \left[ P_{al}'' - \left( \frac{\sigma}{\sigma_{al}} \right)^\alpha \right], \quad (21)$$

where  $\sigma_{al} = \frac{\sigma_{cr}}{c_\sigma}$ ,  $t_{al} = \frac{t_{cr}}{c_t}$ ,  $c_\sigma, c_t$  - safety coefficients.

Out of relation (20) one gets  $t_{cr}^* < t_{cr}$ . The difference  $t_{cr} - t_{cr}^*$  is higher with higher tensile residual stresses. In order to highlight this influence, we examined three samples made of ductile steel, undergoing creep, out of which two were not cold hardened while the other was cold hardened.

One found that in a previously cold hardened sample (deformed with a rate equal to a fraction from the flow level) undergoing creep with a stress which was 10-20% lower than the yield point,  $\sigma_y$ , strain varied sharply [8] after a relatively short time. Creep deformations are, in this case, higher by one order of magnitude to the one observed in the absence of previous cold hardening. The critical duration  $t_{cr}$  for this case is by far lower than in the other cases, which also results from relation (20), established on the basis of the principle of critical energy where the influence of residual stresses is given in the formula of  $P_{cr}''$  (14).

#### Stress loading in corroding media under residual stresses

The uniaxial tensile loading at a temperature which is lower than creep temperature ( $T < T_{creep}$ ), in a corroding environment, of a previously cold hardened sample is a typical problem of effect superposition. This case is solved again by using the principle of critical energy. One writes the total participation in the absence of residual stresses [1, 7].

$$P_T = \left( \frac{\sigma}{\sigma_{cr}} \right)^\alpha + \frac{t'}{t'_{cr}}, \quad (22)$$

where  $\sigma$  is the loading stress,  $\sigma_{cr}$  - critical stress (for example,  $\sigma_{cr} = \sigma_f$  - failure strength in dry air),  $t', t'_{cr}$  - the effective duration and the critical duration respectively, in a corroding environment.

The critical stress for a sample under uniaxial load in a corroding environment results from relation (22) with the condition  $P_T = P_{cr}''$  and is written as

$$\sigma_{cr}^* = \sigma_{cr} \cdot \left( P_{cr}'' - \frac{t'}{t'_{cr}} \right)^{\frac{1}{\alpha}} \quad (23)$$

In the case when the residual stresses are tensile stresses, out of relations (14) and (23) it follows that  $\sigma_{cr}^*$  is lower than in the case when  $\sigma_{res} = 0$ . The occurrence of tensile residual stresses determines a reduction in the time in using the material under load in a corroding medium.

By analogy with relation (20), one obtains the critical time taken until the destruction or the lifetime of the cold hardened material under uniaxial tensile loading in a corroding environment,

$$(t'_{cr})^* = t'_{cr} \cdot \left[ P_{cr}'' - \left( \frac{\sigma}{\sigma_{cr}} \right)^\alpha \right] \quad (24)$$

The residual elongation stresses ( $\delta_{res} = 1$ ) determine a drop in the time taken to reach the critical state, which has also been found experimentally.

#### Fatigue loading in a corroding environment, by considering the residual stresses

Fatigue loading in the corroding environment of a previously cold hardened material is also a problem of effect superposition that may be solved by using the principle of critical energy [1, 6]. The solution is obtained by using the previous results [1]. In relations (22), (23) and

(24) one replaces the term  $\left( \frac{\sigma}{\sigma_{cr}} \right)^\alpha$  with  $\left( \frac{\sigma_a}{\sigma_{-1,p}} \right)^\alpha + \left( \frac{\sigma_m}{\sigma_{m,cr}} \right)^\alpha \cdot \delta_{\sigma_m}$ ,

where:

$\sigma_a$  is stress amplitude;  
 $\sigma_m$  - average stress;

$\sigma_{-1,p}$  - fatigue stress in the presence of stress concentrations;  
 $\sigma_{m,cr}$  - average critical stress ( $\sigma_{m,cr} = \sigma_y$  or  $\sigma_u$ );

$$\delta_{\sigma_m} = \begin{cases} 1, & \text{if } \sigma_m > 0; \\ 0, & \text{if } \sigma_m = 0 \\ -1, & \text{if } \sigma_m < 0. \end{cases} \quad (25)$$

One obtains the following relation, corresponding to the attainment of the critical state,  $P_T = P_{cr}''$ , of a material under fatigue loading in a corroding environment, by considering residual stresses,

$$\left(\frac{\sigma_a}{\sigma_{-1,p}}\right)^\alpha + \left(\frac{\sigma_m}{\sigma_{m,cr}}\right)^\alpha \cdot \delta_{\sigma_m} + \frac{t'}{t_{cr}} = P_{cr}'' \quad (26)$$

The time taken to reach failure (critical state) or the lifetime, in the conditions previously presented derives from relation (26) as

$$t_f' = t_{cr}' \cdot \left[ P_{cr}'' - \left(\frac{\sigma_a}{\sigma_{-1,p}}\right)^\alpha - \left(\frac{\sigma_m}{\sigma_{m,cr}}\right)^\alpha \cdot \delta_{\sigma_m} \right] \quad (27)$$

The residual stresses influence the duration  $t_f'$  up to the moment of failure or the number of loading cycles until the moment of failure,  $N_p$  by means of  $P_{cr}''$ , because  $N_f = (t_f' / t_{cyc})$ , where  $t_{cyc}$  is the duration of a cycle of fatigue.

#### Fatigue loading under creep, in a corroding environment, by considering residual stresses

In this case, superposition involves fatigue loading, thermal loading under creep, the corroding action and the action of residual stresses.

This problem of effects superposition is also solved by using the principle of critical energy and by writing that the total participation is equal to the sum of participation of each action,

$$P_T = \left(\frac{\sigma_a}{\sigma_{-1,p}(T)}\right)^\alpha + \left(\frac{\sigma_m}{\sigma_{m,cr}(T)}\right)^\alpha \cdot \delta_{\sigma_m} + \frac{t}{t_{cr}} + \frac{t'}{t_{cr}} \quad (28)$$

where  $t$  and  $t_{cr} = t_c$  refer to the duration corresponding to the load under creep;  $t'$  and  $t_{cr}'$  refer to the duration of the action of the corroding environment, while the critical stress  $\sigma_{-1,p}(T)$  and  $\sigma_{m,cr}(T)$  and their exponent  $\alpha$  depend on temperature  $T$ .

Out of condition  $P_T = P_{cr}''$  corresponding to the attainment of the critical state, one obtains:

$$\frac{t}{t_{cr}} + \frac{t'}{t_{cr}} = P_{cr}'' - \left(\frac{\sigma_a}{\sigma_{-1,p}(T)}\right)^\alpha - \left(\frac{\sigma_m}{\sigma_{m,cr}(T)}\right)^\alpha \cdot \delta_{\sigma_m} \quad (29)$$

In the fairly frequent case that is often found when both thermal action and the corroding action have the same duration, ( $t=t'$ ) out of relation (29) one obtains the critical duration (the lifetime) in the conditions created by the superposition of loading effects,

$$t_{cr}^* = \frac{t_{cr} \cdot t_{cr}'}{t_{cr} + t_{cr}'} \cdot \left[ P_{cr}'' - \left(\frac{\sigma_a}{\sigma_{-1,p}(T)}\right)^\alpha - \left(\frac{\sigma_m}{\sigma_{m,cr}(T)}\right)^\alpha \cdot \delta_{\sigma_m} \right] \quad (30)$$

Analogously, the duration until the allowable state has been reached is calculated with a relation which is similar to relation (30), by replacing all the critical measures with the corresponding allowable measures. One obtains,

$$t_{al}^* = \frac{t_{al} \cdot t_{al}'}{t_{al} + t_{al}'} \cdot \left[ P_{al}'' - \left(\frac{\sigma_a}{\sigma_{-1,al}(T)}\right)^\alpha - \left(\frac{\sigma_m}{\sigma_{m,al}(T)}\right)^\alpha \cdot \delta_{\sigma_m} \right] \quad (31)$$

where  $P_{al}''$  depends on the residual stresses according to relation (8), while

$$t_{al} = \frac{t_{cr}}{c_t}; \quad t_{al}' = \frac{t_{cr}'}{c_t'}; \\ \sigma_{-1,al}(T) = \frac{\sigma_{-1,p}(T)}{c_{-1}}; \quad \sigma_{m,al}(T) = \frac{\sigma_{m,cr}(T)}{c_m}$$

where  $c_t; c_t'; c_{-1}$  and  $c_m$  are the safety coefficients.

#### Practical examples

In a cylindrical shell (fig. 3) manufactured by cold roll, welded on generators and then subjected to a heat treatment one obtained experimentally [9] that the hoop residual stress in the weld was  $\sigma_{2,res} = 173.2 \div 182.8$  MPa.

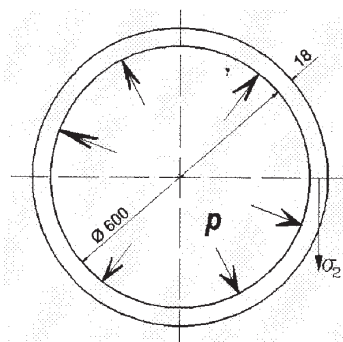


Fig. 3. The shell material has an yield strength of  $\sigma_y(20^\circ) = 384$  MPa and its ultimate strength  $\sigma_u(20^\circ\text{C}) = 525$  MPa

The shell material has an yield strength of  $\sigma_y(20^\circ\text{C}) = 384$  MPa and its ultimate strength  $\sigma_u(20^\circ\text{C}) = 525$  MPa.

In the case when under working conditions, the shell is subjected to an internal pressure, the residual tensile stresses are not favourable ( $\delta_{res} = 1$ ).

In this case, the tensile residual stresses are dangerous to the mechanical strength of the shell.

The critical participation in isothermal loading, in this case, according to relation (14), is:

$$P_{cr}'' = P_{cr} - \left(\frac{182,8}{384}\right)^2 = P_{cr} - 0.2266.$$

If one allows, at best, that  $P_{cr} = 1$  then the occurrence of residual stresses leads to a decrease in the carrying capacity of the shell by 22.66%.

The allowable participation under isothermal loading, by considering the residual stresses, according to relation (18) is written as:

$$P_{al}'' = 1 - \left(\frac{182,8}{256}\right)^2 = 0,49, \\ \sigma_{res,al} = \frac{\sigma_{res,cr}}{c_\sigma} = \frac{384}{1,5} = 256 \text{ MPa}.$$

One finds a 51% decrease in the carrying capacity with respect to the allowable state that indicates that overlooking the residual stresses in the calculation of strength may be dangerous.

### Conclusions

By using the principle of critical energy, it has become possible to superpose the effects found in loaded materials with non-linear behaviour. This has also allowed the calculation of the effect of residual stresses resulting from the manufacturing processes upon the strength and upon the lifetime.

The calculation of strength of shells based on the principle of critical energy is made in two stages:

-the calculation of total energy participation with respect to the critical state,  $P_T$ , or the calculation of the total energy participation, with respect to the allowable state,  $P_T^*$  ;

-the comparison of the total energy participation  $P_T$ , with the corresponding critical energy participation, by considering the thermal effect, as well as the effect of the residual stresses. If:

$P_T < P_{cr}''$ , the critical state is not reached;

$P_T \geq P_{cr}''$  the critical state is reached or exceeded.

If the limiting state is the allowable state, one compares the total specific energy participation with respect to the allowable state,  $P_T^*$ , with the  $P_a^*$  value. If,

$P_T \leq P_a''$ , the loading state is allowable;

$P_T^* > P_a''$ , the loading state is not allowable.

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